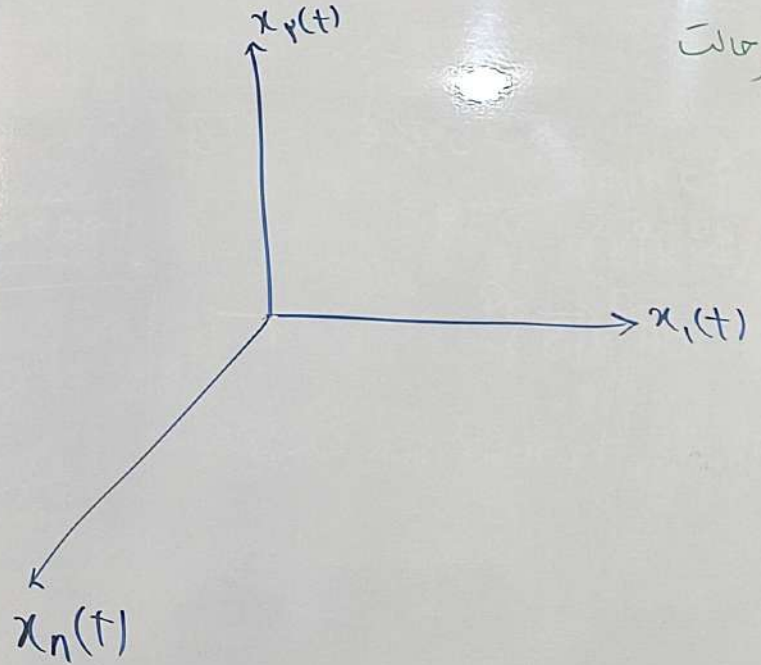
فصل چهارم: معادلات حالت

$x_1(t),\ x_2(t),\ \cdots,\ x_n(t)$

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

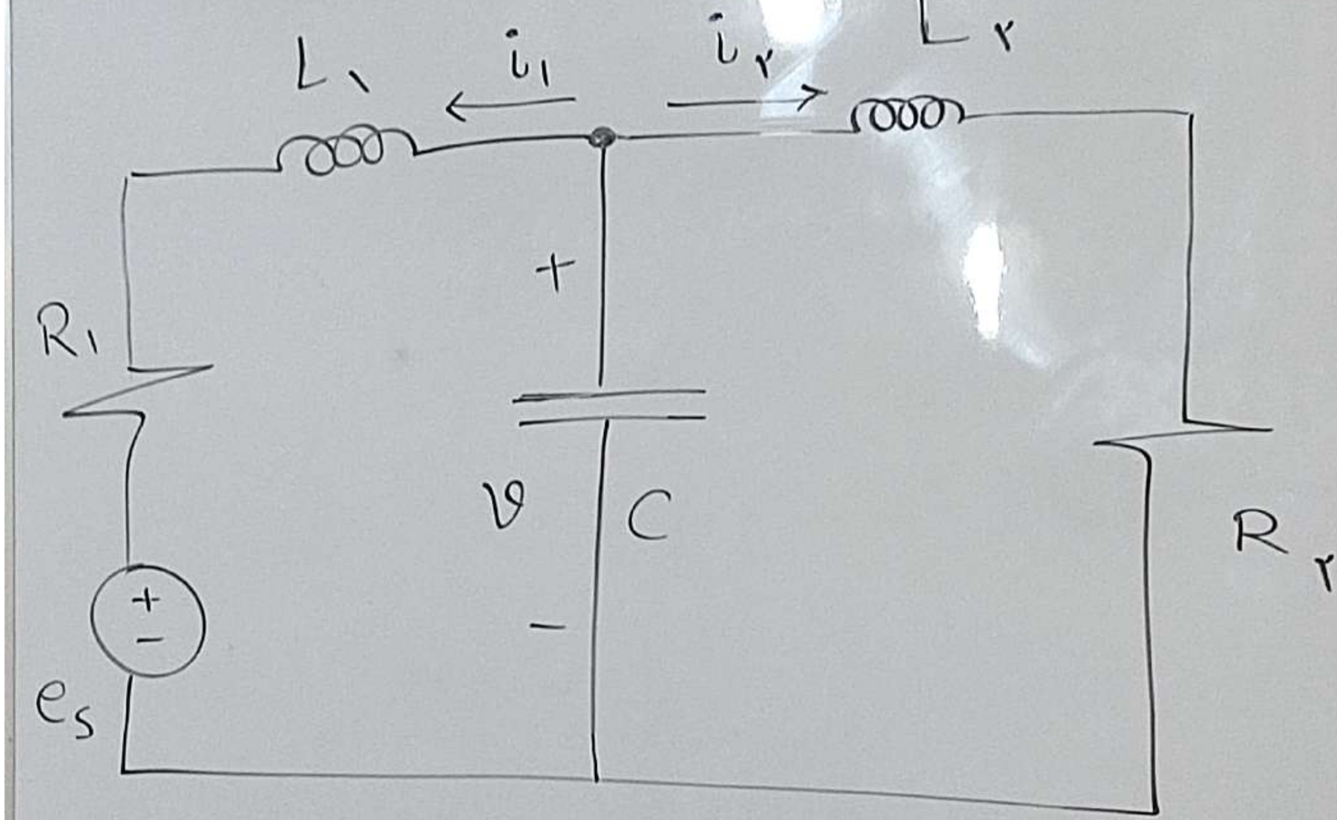


مشتق متغیر حالت

$$\dot{x}(t)=f(x,u,t)$$

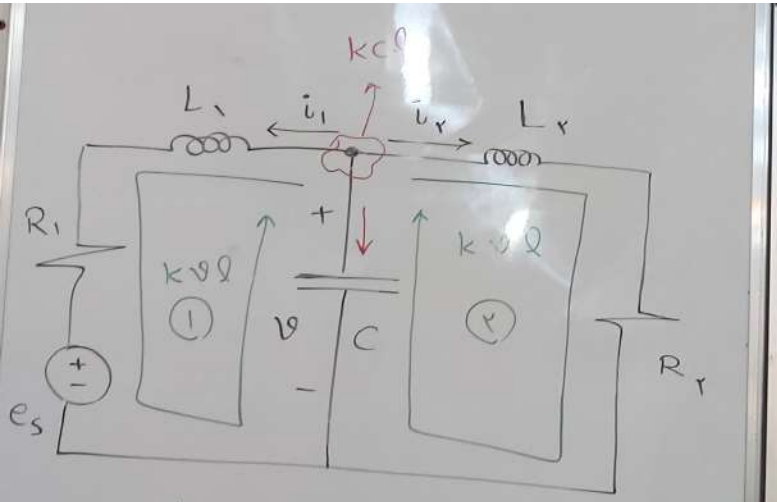
- $\dot{x}(t)$: مشتق متغیر حالت
- x: خود متغیر حالت
- u: منابع
- t: زمان

$$\dot{x}(t)=\frac{dx(t)}{dt}$$



فیزور

لاپلاس



KVL ②) $L_2 \frac{di_2}{dt} + R_2 i_2 - v = 0$

$L_2 \frac{di_2}{dt} = v - R_2 i_2$

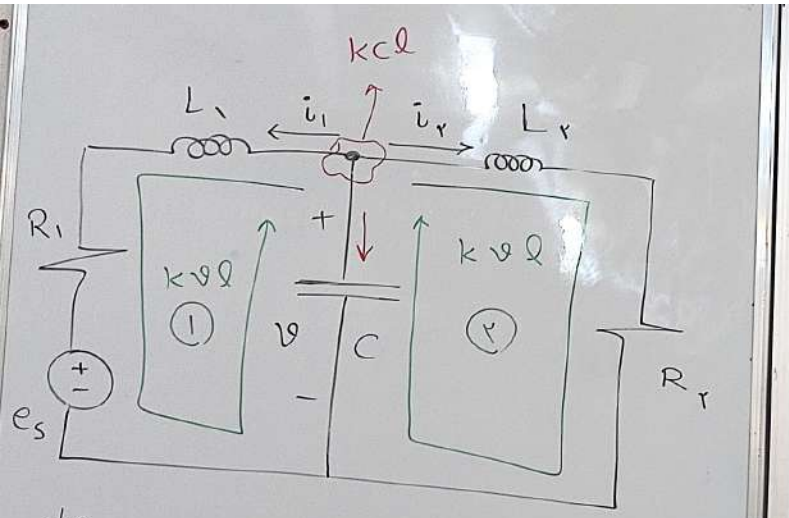
KCL) $C \frac{dv}{dt} + i_1 + i_2 = 0 \Rightarrow C \frac{dv}{dt} = -i_1 - i_2$

$\frac{dv}{dt} = -\frac{1}{C} i_1 - \frac{1}{C} i_2$ ①

KVL ①) $L_1 \frac{di_1}{dt} + R_1 i_1 + e_s - v = 0 \Rightarrow L_1 \frac{di_1}{dt} = v - R_1 i_1 - e_s$

$\frac{di_1}{dt} = \frac{1}{L_1} v - \frac{R_1}{L_1} i_1 - \frac{1}{L_1} e_s$ ②

$\frac{di_2}{dt} = \frac{1}{L_2} v - \frac{R_2}{L_2} i_2$ ③



$$\frac{dv}{dt} = -\frac{1}{C} i_1 - \frac{1}{C} i_2$$

$$\frac{di_1}{dt} = \frac{1}{L_1} v - \frac{R_1}{L_1} i_1 - \frac{1}{L_1} e_s$$

$$\frac{di_2}{dt} = \frac{1}{L_2} v - \frac{R_2}{L_2} i_2$$

$$\left(\frac{d}{dt}\right) \begin{bmatrix} v \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} & -\frac{1}{C} \\ \frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} v \\ i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L_1} \\ 0 \end{bmatrix} e_s$$

اف / $\frac{d}{dt} = D$

ب / $\frac{d}{dt} = j\omega$

ج / $\frac{d}{dt} = s$

$$D = j\omega = s$$

سیستم‌های LTI:

Linear Time Invarient

$$\dot{x}(t) = A x + B u$$

$$\dot{x}(t) = f(x, u, t)$$

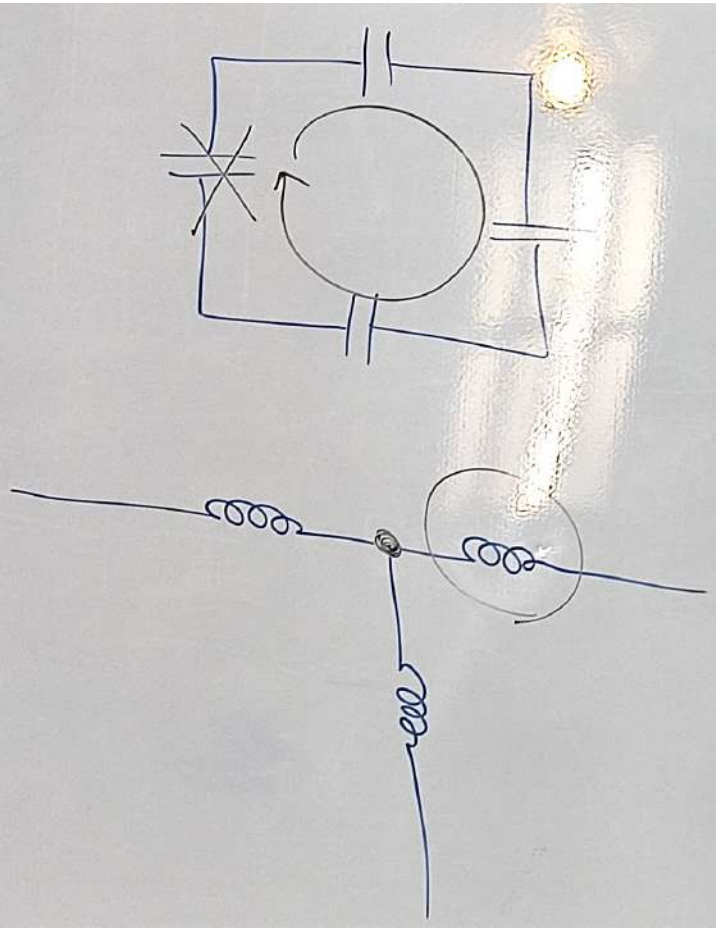
$$\underbrace{\frac{d}{dt}\begin{bmatrix} v \\ i_1 \\ i_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{C} & -\frac{1}{C} \\ \frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v \\ i_1 \\ i_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ -\frac{1}{L_1} \\ 0 \end{bmatrix}}_{B} \underbrace{e_s}_{u}$$

الف) $\frac{d}{dt} = D$

ب) $\frac{d}{dt} = j\omega$

ج) $\frac{d}{dt} = s$

$$D = j\omega = s$$



الگوریتم معادلات حالت :

① انتخاب متغیرهای حالت

② انتخاب درخت مناسب

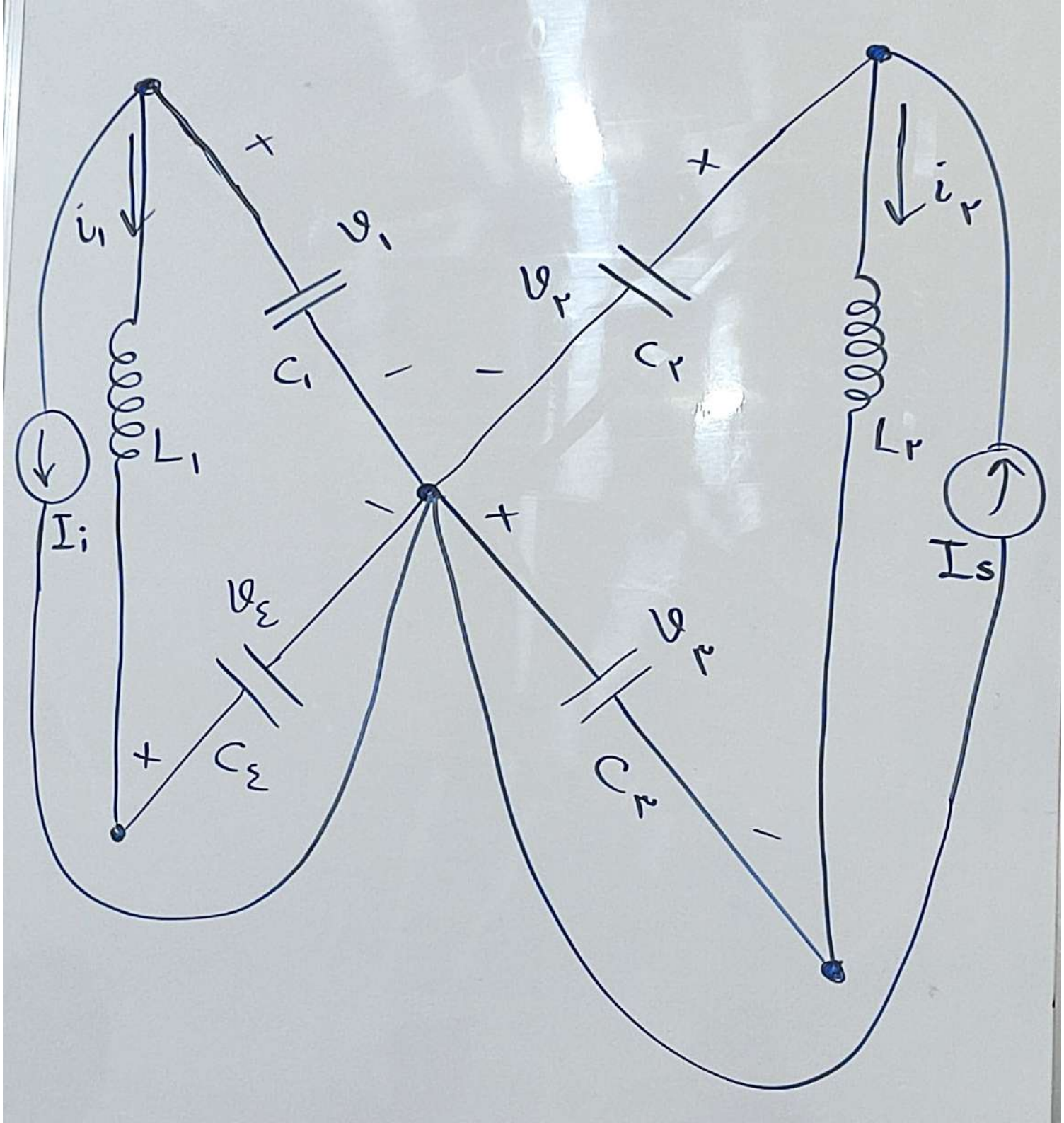
الف حلقه خازنی

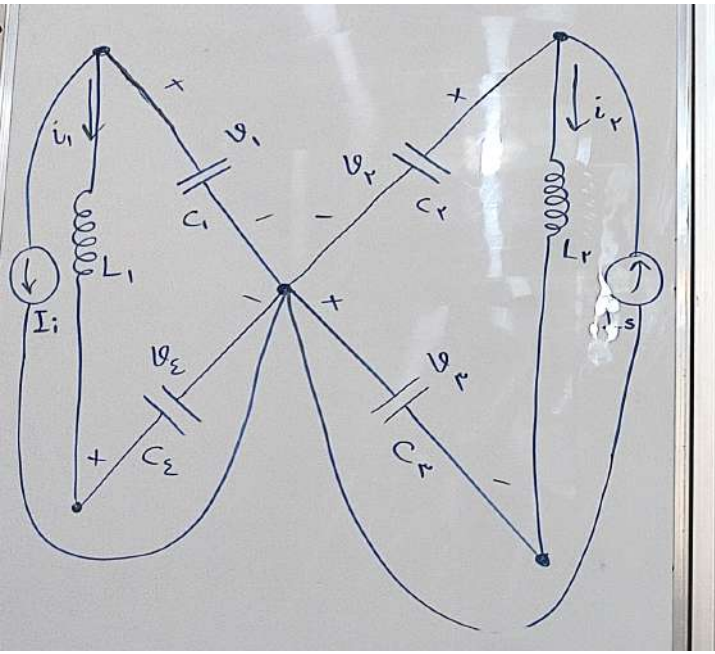
ب کات ست سلفی

③ کات ست اساسی

④ حلقه اساسی

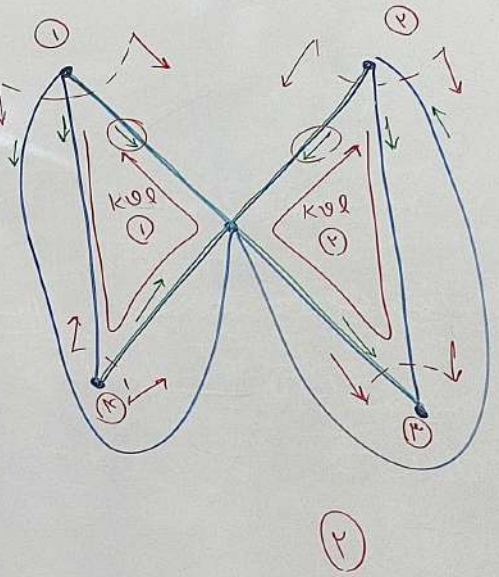
⑤ معادله کلی


i_1
L_1
I_i
C_1
v_1
C_2
v_2
i_2
L_2
I_s
C_3
v_3
C_4
v_4
+
−



(۱)

$$x(t) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_1 \\ i_2 \end{bmatrix}$$



(۲)